Solutions: Chapter 3

Exercise 1

Show that the definition of the 'cause-specific hazard' leads to

$$P(T \leq t, X_T = 1) = \int_0^t P(T \geq u)\alpha_{01}(u)du.$$

Proof. Definition of the cause-specific hazard:

$$\begin{aligned}\alpha_{01}(t)dt &= P(T \in dt, X_T = 1 | T \geq t) \\ &= \frac{P(T \in dt, X_T = 1)}{P(T \geq t)}.\end{aligned}$$

Hence,

$$\begin{aligned}P(T \geq t)\alpha_{01}(t)dt &= P(T \in dt, X_T = 1) \\ &= \frac{d}{dt} P(T \leq t, X_T = 1).\end{aligned}$$

Thus

$$\int_0^t P(T \geq u)\alpha_{01}(u)du = P(T \leq t, X_T = 1)$$

□

Show that $P(T \leq t) = 1 - \exp(-A_{0\cdot}(t)) = \int_0^t P(T \geq u)\alpha_{0\cdot}(u-)du$.

Proof. Due to Chapter 2, exercise 1,

$$P(T \leq t) = 1 - \exp(-\int_0^t \alpha_{0\cdot}(u)du)$$

and $\int_0^t \alpha_{0\cdot}(u)du = A_{0\cdot}(t)$.
Moreover,

$$P(T \geq t)\alpha_{0\cdot}(t)dt = P(T \in dt)$$

Integrating both sides gives

$$\int_0^t P(T \geq u)\alpha_{0\cdot}(u)du = P(T \leq t)$$

□

Alternatively, consider a degenarated competing risks model with only one competing risks.

Exercise 2

Function to simulate a competing risks data.
Input:

`n`**:** sample size

`w.shape` **and** `w.scale`**:** parameters for the weibull cause-specific hazard $\alpha_{01}(t)$

`max.int` Parameter for the numerical inversion. Can stay as it is

Output:
A dataframe in the format required by **etm** (`id, from, to, time`)

```
> sim.chap3.exo2 <- function(n, w.shape, w.scale,
+                            max.int = 500) {
+
+     temp <- function(x, y) {
+         return(1.8 * log(x + 2) + (x / w.scale)^w.shape + y)
+     }
+
+     inc <- 1; n.failure <- 0
+     stime <- numeric(n)
+     while (inc <= n) {
+         u <- runif(1)
+         if (temp(0, log(1 - u)) * temp(max.int, log(1 - u)) < 0) {
+             res <- uniroot(temp, c(0, max.int), tol = 0.0001, y = log(1 - u))
+             stime[inc] <- res$root
+             inc <- inc + 1
+         }
+         else {
+             n.failure <- n.failure + 1
+         }
+     }
+
+     probs <-  ((w.shape/w.scale^w.shape) * stime^(w.shape - 1)) /
+         ((w.shape/w.scale^w.shape) * stime^(w.shape - 1) + (1.8 / (stime + 2)))
+
+     to <- rbinom(n, 1, prob = probs)
+     to <- ifelse(to == 0, 2, 1)
+     time <- stime
+
+     from <- 0; id <- seq_along(time)
+     dat <- data.frame(id, time, from, to)
+     return(dat)
+ }
```

Creation of a data

```
> set.seed(43201)
> dat.exo2 <- sim.chap3.exo2(1000, 3, 2)
```

Cumulative incidence functions using the **etm** package

```
> ## matrix of logical defining the possible transitions
> tra.cp <- matrix(FALSE, nrow = 3, ncol = 3)
> tra.cp[1, 2:3] <- TRUE
> require(etm)
> cifs <- etm(dat.exo2, c("0", "1", "2"), tra.cp, NULL, 0)
```

Proportions:

```
> props <- table(dat.exo2$to) / nrow(dat.exo2)
> props
```

```
    1     2
0.335 0.665
```

which then can be compared the cumulative incidence estimates

```
> trprob(cifs, "0 1", floor(max(cifs$time)))
```

```
[1] 0.33
```

```
> trprob(cifs, "0 2", floor(max(cifs$time)))
```

```
[1] 0.665
```

Exercise 3

- For (a), the cause-specific hazards are

$$\begin{aligned}\alpha_{0j}(t) &= -\frac{d}{dt}\log Q(t,t) \quad \text{(partial derivative w.r.t to } t_j \text{ evaluated at } t_j = \text{t, } j = 1,2)\\ &= \gamma_j(1+\gamma_{12}\exp(\gamma_{12}(\gamma_1+\gamma_2)t)).\end{aligned} \tag{1}$$

  And the marginal hazards

$$\begin{aligned}\alpha^*_{01}(t) &= -\frac{d}{dt}\log Q(t,0)\\ &= \gamma_1(1+\gamma_{12}\exp(\gamma_1\gamma_{12}t)),\\ \alpha^*_{02}(t) &= -\frac{d}{dt}\log Q(0,t)\\ &= \gamma_2(1+\gamma_{12}\exp(\gamma_2\gamma_{12}t)).\end{aligned}$$

- For (b), the cause-specific hazards equal the marginal hazards and are equal to (1).

As, for model (b), the cause-specific hazards and the marginal hazards are the same, model (b) is an independent latent failure time model.